The fate of the α-vacuum

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Overview

- **The vacua of de Sitter space**

- **The linear divergences of an interacting theory in an α-vacuum**

- **Taming the divergences of the α-vacuum**

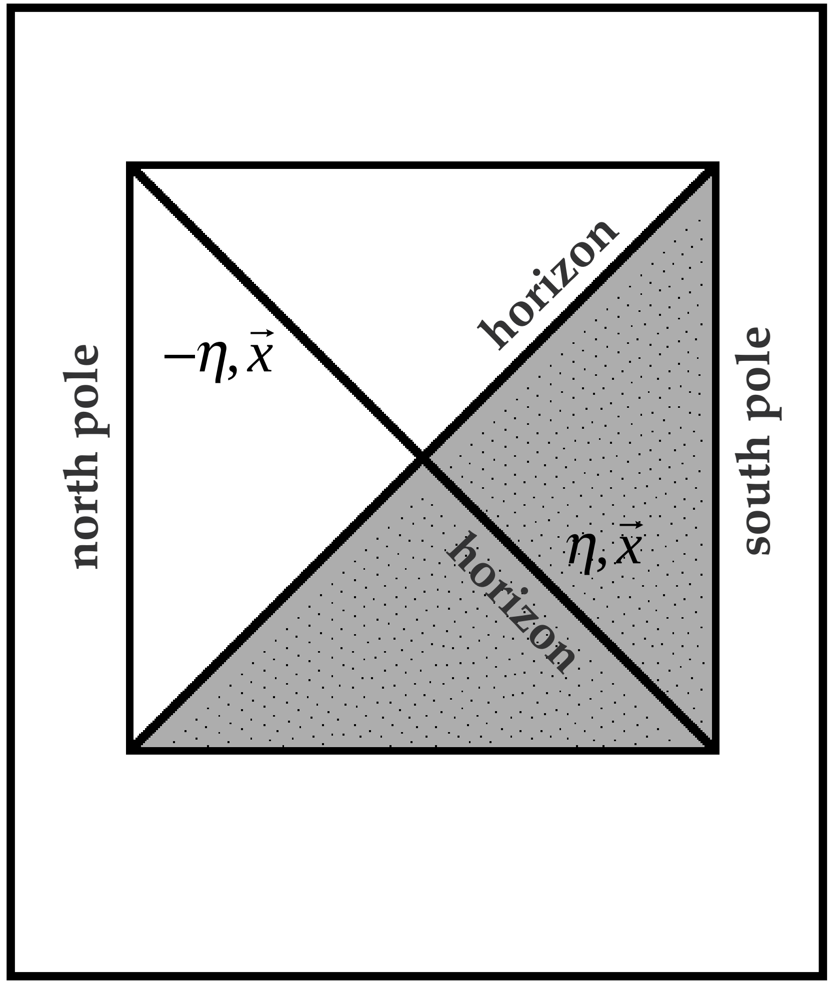
- **Truncated α-vacua and the CMB (next talk)**

The geometry of de Sitter space

- The cosmological importance of de Sitter space challenges us to understand properly quantum field theory in a de Sitter background
- **no global time-like Killing vector**
- Conformally flat coordinates:

$$\boxed{ds^2 = \frac{d\eta^2 - d\vec{x}\cdot d\vec{x}}{\eta^2}}$$

What are the de Sitter invariant vacua?



The vacua of de Sitter space

- One vacuum of de Sitter space has special properties:
  - **thermal,**
  - **becomes the flat vacuum at short distances, …**

Consider a free scalar field:

$$\Phi(\eta,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3}\left[U_k^E(\eta)e^{i\vec{k}\cdot\vec{x}}a_{\vec{k}}^E + U_k^{E*}(\eta)e^{-i\vec{k}\cdot\vec{x}}a_{\vec{k}}^{E\dagger}\right]$$

defines the vacuum: $a_{\vec{k}}^E\left|E\right\rangle = 0$ $\qquad U_k^E(\eta) = \frac{\sqrt{\pi}}{2}\eta^{3/2}H_\nu^{(2)}(k\eta)$

Mottola, PRD 31, 754 (1985)
Allen, PRD 32, 3136 (1985)

- The α-vacuum

$$a_{\vec{k}}^\alpha = N_\alpha\left[a_{\vec{k}}^E - e^{\alpha^*}a_{-\vec{k}}^{E\dagger}\right] \qquad N_\alpha = \left(1-e^{\alpha+\alpha^*}\right)^{-1/2}$$

$$\mathrm{Re}\,\alpha < 0$$

defines a new vacuum: $a_{\vec{k}}^\alpha\left|\alpha\right\rangle = 0$

UV divergences of an interacting theory in the α-vacuum

Trouble with the α-vacua?

Perturbation theory in the α-vacua:

- Einhorn and Larsen found pinched singularities at one-loop order

  hep-th/0209159

- Banks and Mannelli showed the α-vacua need non-local counterterms

  hep-th/0209113

Variants of the α-vacua can be renormalized:

- Place sources at x and x_A
  - Goldstein and Lowe

    hep-th/0302050, hep-th/0308135
  - Einhorn and Larsen

    hep-th/0305056

The approach:

- **First study the original α-vacuum propagators**
- **Examine the variants later**

$$G_\alpha^F(x,x') \propto \frac{1}{Z-1-i\varepsilon} + \frac{e^{\alpha+\alpha^*}}{Z-1+i\varepsilon} - \frac{e^{\alpha}}{Z+1-i\varepsilon} - \frac{e^{\alpha^*}}{Z+1+i\varepsilon}$$

de Sitter invariant distance

- **Evolve matrix elements**
  - **do not ask S-matrix questions**
  - **Suppose that the system is in an α-vacuum at η_0 and evolve forward**

The fate of the α-vacuum in an interacting theory

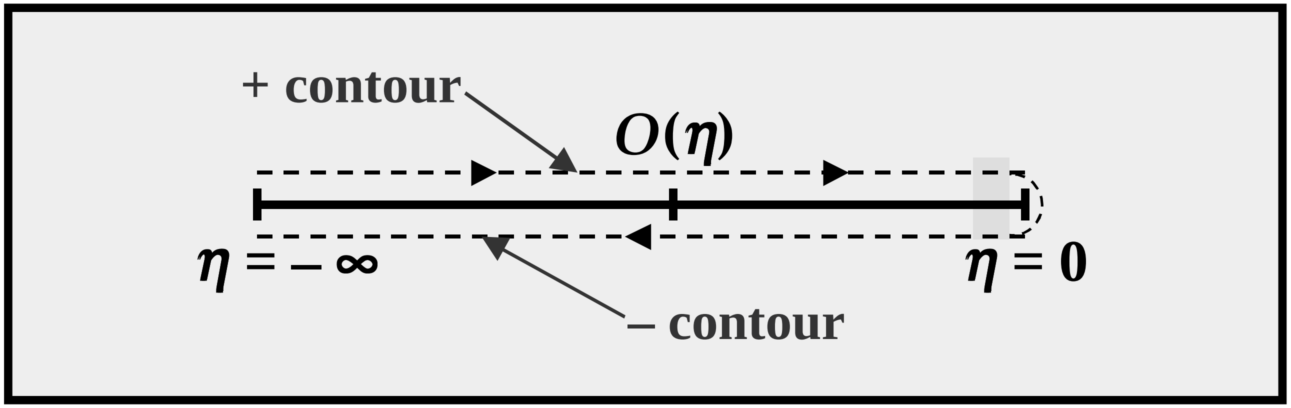
- When we evolve the quantum theory with interactions,
  - we discover a linear divergence in the one-loop corrections
  - which cannot be cancelled by a mass counterterm

- We use a quantization formalism in which the pinch singularities do not appear
  - cf. pinch singularities in thermal field theory

- As a check we find that the theory is renormalizable when
  - $\alpha \to -\infty$ (Euclidean vacuum) with interaction, or
  - $\alpha \neq -\infty$ but with no interactions

The Schwinger-Keldysh formalism

- As the background is time-dependent, it is dangerous to ask about the asymptotic states



- We use a quantization procedure that allows a time-dependent evolution

Schwinger, J. Math. Phys 2, 407 (1961)
Keldych, JETP 20, 1018 (1965)

- Effectively double the field content of the theory:

$$H_I(\Phi) \to H_I(\Phi^+) - H_I(\Phi^-)$$

For the Closed Time Contour

- **Double the vertices:**
  - **use Φ^+ and Φ^- vertices**
- **Four propagators:**
  - **G^{++}, G^{+-}, G^{-+}, G^{--}**
- **Events on the + contour occur before those on the – contour**

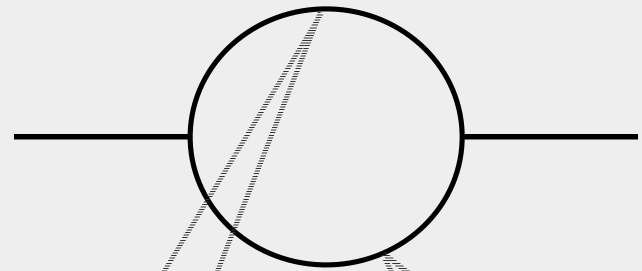
Divergences in the α-vacuum

- Loop propagator
- Power counting
- Phase interference
- Possible divergences in
  - two-propagator loops
  - three-propagator loops

$$\int^{\Lambda} d^3\vec{p} \prod_{i=1}^{n} G^{\alpha}_{\vec{p}-\vec{k}_i}(\eta_i, \eta_{i+1}) \approx \int^{\Lambda} \frac{dp}{p^{n-2}}$$

***n*=2 linearly divergent**

***n*=3 logarithmically divergent**



α-Wightman function:

$$G^{>}_{\vec{p}-\vec{k}_i}(\eta_i, \eta_{i+1}) = iN_{\alpha}^2 \frac{\eta_i \eta_{i+1}}{2\left|\vec{p}-\vec{k}_i\right|} \Big[e^{-i\left|\vec{p}-\vec{k}_i\right|(\eta_i - \eta_{i+1})} + e^{\alpha+\alpha^*} e^{i\left|\vec{p}-\vec{k}_i\right|(\eta_i - \eta_{i+1})} - e^{\alpha} e^{i\left|\vec{p}-\vec{k}_i\right|(\eta_i + \eta_{i+1})} - e^{\alpha^*} e^{-i\left|\vec{p}-\vec{k}_i\right|(\eta_i + \eta_{i+1})} \Big]$$

Evolution of the number operator

- Example:
  - the number of Euclidean particles in the α-vacuum
  - in α-state at η_0
  - Φ^3 interaction
- Evaluate to one-loop order

$$N^E_{\vec{k}}(\eta) = a^{E\dagger}_{\vec{k}}(\eta) a^{E\dagger}_{\vec{k}}(\eta)$$

induced by H_0

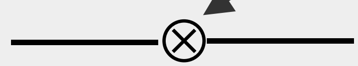
$$\langle \alpha(\eta) | N^E_{\vec{k}}(\eta) | \alpha(\eta) \rangle = \text{to } \mathcal{O}(\lambda^2)$$

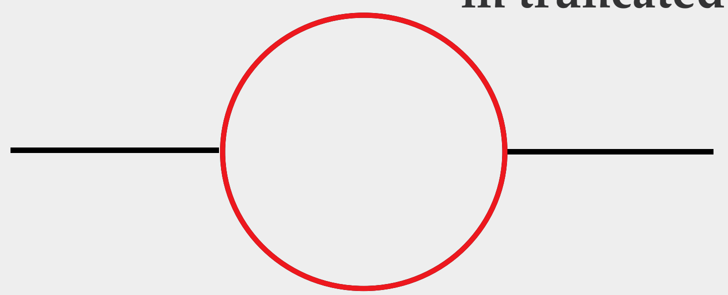
$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \tfrac{1}{2} m^2 \Phi^2 - \rho\Phi - \tfrac{1}{2}\delta m^2 \Phi^2 - \tfrac{1}{6}\lambda \Phi^3$$

remove log divergence

— in Euclidean limit

— in truncated α-vacuum

$$\langle \alpha(\eta) | N^E_{\vec{k}}(\eta) | \alpha(\eta) \rangle = \mathcal{O}(\lambda^0) +$$ 

$+$  $+ \quad \cdots$

Origin of the divergence

- Define an 'initial occupation number'

$$\langle\alpha|a_{\vec{k}}^{E\dagger}a_{\vec{k}}^{E}|\alpha\rangle V^{-1} = N_\alpha^2 e^{\alpha+\alpha^*} \equiv n_{\vec{k}}^{\alpha}$$

independent of *k*

- The α-dependent coefficient of the divergence is thus

$$\dot{N}_{\alpha,\vec{k}}^{E} = -\frac{\lambda^2}{k\eta}\frac{V}{64\pi^3}\frac{1}{k\eta_0}\int_{\eta_0}^{\eta}\frac{d\eta'}{\eta'}\int\frac{d^3\vec{p}}{p|\vec{p}-\vec{k}|} \to \frac{p^2dp}{p^2}$$

$$\times\left[\underline{(n_k^\alpha+1)(n_p^\alpha+1)n_{|p-k|}^\alpha} - \underline{n_k^\alpha n_p^\alpha(n_{|p-k|}^\alpha+1)}\right]$$

$$\times\sin\left[\left[p+k-|\vec{p}-\vec{k}|\right](\eta-\eta')\right]$$

$$+\cdots$$

linearly divergent

p*-independent at large *p

If this were a thermal system these n_p^α s would have a Boltzmann suppression at large *p*

However, an α-state is populated to arbitrarily high momenta

Taming the α-vacuum

What went wrong with the α-vacuum?

- Since the Euclidean vacuum matches with the flat vacuum, it is reasonable to define the usual time-ordering
- But is it the correct prescription for the α-vacuum?
  - antipodinal information
  - interference
- Can we generalize the idea of time-ordering
  - distinct from Euclidean case
  - has a good $\alpha \to -\infty$ limit

$$\langle \alpha | T(\Phi(x)\Phi(x')) | \alpha \rangle$$

$$\overset{?}{=} \Theta(t-t')G_\alpha(x,x') + \Theta(t'-t)G_\alpha(x',x)$$

no t_A

$$G_\alpha(x,x')$$

$$= N_\alpha^2 \big[G_E(x,x') + e^{\alpha+\alpha^*} G_E(x',x) + e^{\alpha} G_E(x_A,x') + e^{\alpha^*} G_E(x',x_A) \big]$$

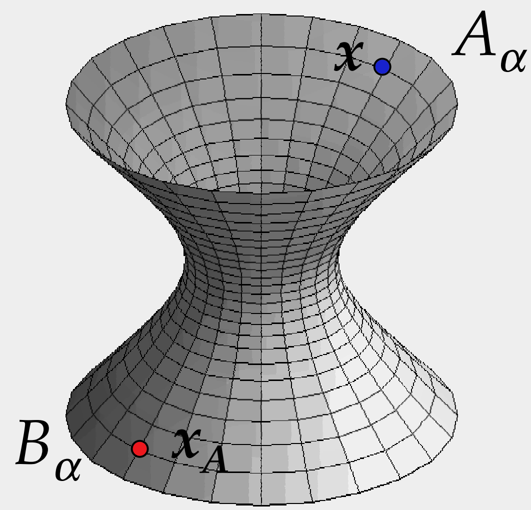
Propagation in the α-vacuum

- Define a new time-ordering for the α-vacuum to remove constructive interference in products of propagators

$$\lim A_\alpha \xrightarrow{\alpha \to E} 1$$
$$\lim B_\alpha \xrightarrow{\alpha \to E} 0$$

- Up to constants, we obtain an essentially unique form
  - contains two point sources
- **Goldstein and Lowe, hep-th/0302050, hep-th/0308135**
- **Einhorn and Larsen, hep-th/0305056**

$$\begin{aligned}\langle \alpha | T_\alpha(\Phi(x)\Phi(x')) | \alpha \rangle \\ &= -iA_\alpha G_E(x,x') - iB_\alpha G_E(x_A,x') \\ &= -i\,G_\alpha^F(x,x')\end{aligned}$$



Time ordering in the α-vacuum

- Note that the time-ordering operator acts on the fields
- To disentangle the inherent correlated behavior at the antipodes, T_α also depends on the antipodes

$$
\begin{aligned}
&T_\alpha\big(\Phi(x)\Phi(x')\big) \\
&\quad = \Theta_\alpha(t,t')\Phi(x)\Phi(x') \\
&\qquad + [\Theta_\alpha(t',t)]^* \Phi(x')\Phi(x) \\
&\qquad + \Theta_\alpha^A(t_A,t')\Phi(x_A)\Phi(x') \\
&\qquad + \left[\Theta_\alpha^A(t',t_A)\right]^* \Phi(x')\Phi(x_A)
\end{aligned}
$$

- It is useful to have a path integral definition for the field theory

$$
\begin{aligned}
\Theta_\alpha(t,t') \equiv \frac{1}{1-e^{2\alpha}} \Big[& A_\alpha \left[\Theta(t-t') + e^{2\alpha}\Theta(t'-t)\right] \\
& - B_\alpha e^{\alpha}\left[\Theta(t_A - t') + \Theta(t' - t_A)\right]\Big]
\end{aligned}
$$

The path integral: free theory

- Since the propagator contains two sources, let us define the generating functional to have two currents:

$$\begin{aligned}\langle\alpha|T_\alpha(\Phi(x)\Phi(x'))|\alpha\rangle &= -iA_\alpha G_E^F(x,x') - iB_\alpha G_E^F(x_A,x') \\ &= -i\,G_\alpha^F(x,x')\end{aligned}$$

$$W_0^\alpha[J] = \int \mathcal{D}\Phi\, e^{i\int d^4x\sqrt{-g}\,[\mathcal{L}_0 + (a_\alpha J(x) + b_\alpha J(x_A))\Phi(x)]}$$

- Fix a_α and b_α by differentiating with respect to the current
- Complete the square in W_0^α

$$\begin{aligned}&\left[-i\tfrac{\delta}{\delta J(x)}\right]\left[-i\tfrac{\delta}{\delta J(x')}\right]W_0^\alpha[J]\Big|_{J=0} \\ &\quad = \langle\alpha|T_\alpha(\Phi(x)\Phi(x'))|\alpha\rangle \\ &\quad A_\alpha = a_\alpha^2 + b_\alpha^2 \qquad B_\alpha = 2a_\alpha b_\alpha\end{aligned}$$

The path integral: local interactions

$$W^{\alpha}[J] = \int \mathscr{D}\Phi\, e^{i\int d^4x \sqrt{-g}\,[\mathscr{L} + (a_\alpha J(x) + b_\alpha J(x_A))\Phi(x)]}$$

$$\mathscr{L}[\Phi(x)] = \mathscr{L}_0[\Phi(x)] + \mathscr{L}_I[\Phi(x)]$$

- Express W^{α} in terms of W_0^{α}
- Define a new functional derivative
- An n-point Green's function contains three types of propagators
  - Alpha (both external)
  - Mixed (one point external)
  - Euclidean (internal)

$$G_\alpha^n(x_1,\ldots,x_n) = \left[-i\frac{\delta}{\delta J(x_1)}\right]\cdots\left[-i\frac{\delta}{\delta J(x_n)}\right]$$

$$\times N\, e^{i\int d^4x \sqrt{-g}\,\mathscr{L}_I\left[-i\frac{\delta}{\delta \mathrm{J}(x)}\right]} W_0^{\alpha}[J]$$

$$\left[-i\frac{\delta}{\delta \mathrm{J}(x)}\right]\left[-i\frac{\delta}{\delta \mathrm{J}(x')}\right] W_0^{\alpha}[J]\Big|_{J=0} = -i\,G_E^F(x,x')$$

the theory is renormalizable

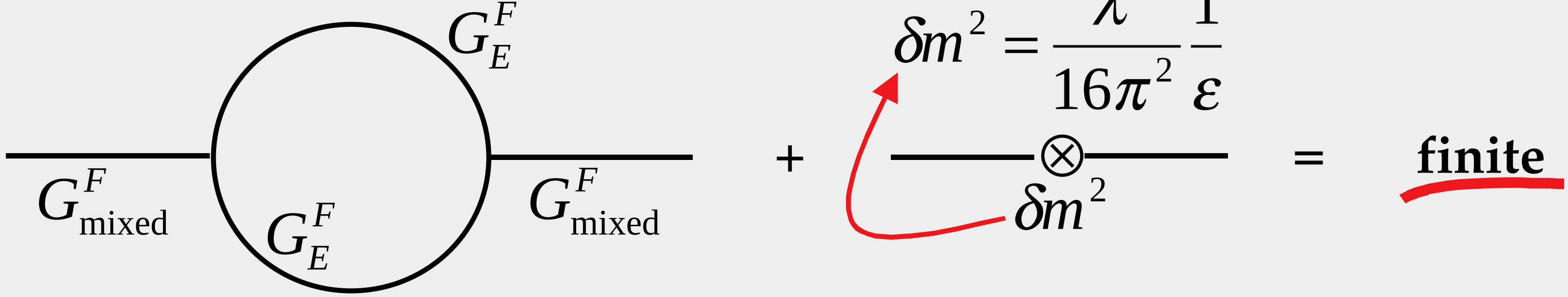
Local interactions: renormalizability

$$W^{\alpha}[J] = \int \mathscr{D}\Phi \, e^{i\int d^4x\sqrt{-g}\,[\mathscr{L} + (a_\alpha J(x) + b_\alpha J(x_A))\Phi(x)]}$$

$$\mathscr{L}[\Phi(x)] = \tfrac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \tfrac{1}{2}m^2\Phi^2$$
$$-\rho\Phi - \tfrac{1}{2}\delta m^2\Phi^2 - \tfrac{1}{6}\lambda\Phi^3$$

- Only Euclidean-vacuum propagators appear in the loop
- The loop produces a logarithmic divergence which is cancelled by δm^2

$$\delta m^2 = \frac{\lambda^2}{16\pi^2}\frac{1}{\varepsilon}$$



The path integral: antipodinal interactions

$$W^{\alpha}[J] = \int \mathcal{D}\Phi\, e^{i\int d^4x\sqrt{-g}\,[\mathcal{L} + (a_\alpha J(x) + b_\alpha J(x_A))\Phi(x)]}$$

$$\mathcal{L} = \mathcal{L}_0[\Phi(x)] + \mathcal{L}_I[a_\alpha \Phi(x) + b_\alpha \Phi(x_A)]$$

- Local interactions are renormalizable because only $G_E{}^F$ appears in loops
- Was this necessary?
- Recall the original α-vacuum
  - Euclidean
  - Double-source α
- Antipodinal interactions

$$G^n_\alpha(x_1,\ldots,x_n) = \left[-i\frac{\delta}{\delta J(x_1)}\right]\cdots\left[-i\frac{\delta}{\delta J(x_n)}\right] \times N\, e^{i\int d^4x\sqrt{-g}\,\mathcal{L}_I\left[-i\frac{\delta}{\delta J(x)}\right]} W_0^{\alpha}[J]$$

$$\left[-i\frac{\delta}{\delta J(x)}\right]\left[-i\frac{\delta}{\delta J(x')}\right] W_0^{\alpha}[J]\Big|_{J=0} = -i\,G^F_\alpha(x,x')$$

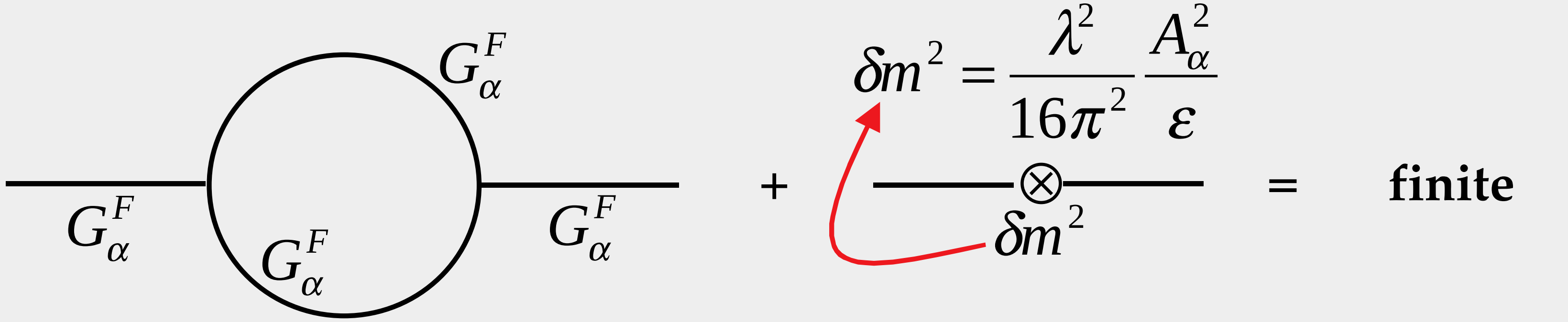
Antipodinal interactions: renormalizability

$$W^{\alpha}[J] = \int \mathcal{D}\Phi\, e^{i\int d^4x\sqrt{-g}\,[\mathcal{L} + (a_\alpha J(x) + b_\alpha J(x_A))\Phi(x)]}$$

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \tfrac{1}{2}m^2\Phi^2 - \rho[a_\alpha\Phi(x) + b_\alpha\Phi(x_A)]$$
$$-\tfrac{1}{2}\delta m^2[a_\alpha\Phi(x) + b_\alpha\Phi(x_A)]^2 - \tfrac{1}{6}\lambda[a_\alpha\Phi(x) + b_\alpha\Phi(x_A)]^3$$

- Only α-vacuum propagators appear in diagrams
- This loop produces a logarithmic divergence too which is cancelled by δm^2

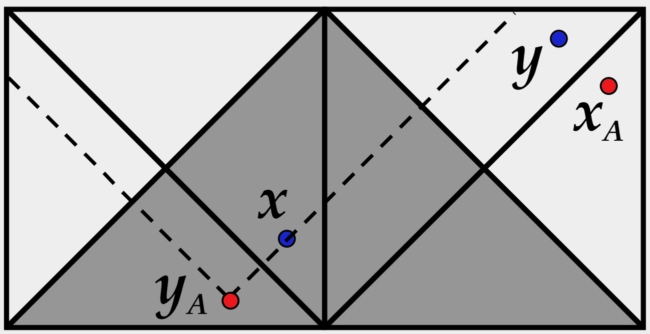
$$\delta m^2 = \frac{\lambda^2}{16\pi^2}\frac{A_\alpha^2}{\varepsilon}$$



Further tests

- Although the propagator contains a peculiar piece which depends on the antipode, the theory is
  - causal
  - renormalizable in the self-energy
- It is non-local, but in a very constrained form
  - global coordinates
  - inflationary patch
- Do pathologies appear elsewhere?
  - e.g. vertex corrections?

$$G^F_\alpha(x,x') = -\frac{iA_\alpha}{8\pi^2}\frac{1}{Z(x,x')-1-i\varepsilon} + \frac{iB_\alpha}{8\pi^2}\frac{1}{-Z(x_A,x')+1-i\varepsilon}$$



Conclusions

- The α-vacuum
  - The one loop corrections are linearly divergent for the α-vacuum
  - These cannot be removed by a mass counterterm
  - Euclidean vacuum loops can be renormalized
- An α-vacuum with two sources
  - Our definition for the α-propagator is based on flat space intuition
  - Use renormalizability to guide us
  - Self-energy graphs are no longer linearly divergent
    - local and antipodinal interactions
- Loops and trans-planckian physics
  - Loop effects receive a Λ/H enhancement
    - the next talk will describe this result